ALGEBRAIC EXPRESSIONS

First revise the work done in Grade 8 and make sure that you are able to do all the examples in the Grade 8 study guide.

1. MULTIPLY MONOMIALS BY POLYNOMIALS

Last year you determined the product of a monomial and a binomial by multiplying each term of the binomial by the monomial.
For example:

$-2x(x-2) = -2x^2 + 4x$ $\quad [-2x \times x = 2x^2 \text{ and } -2x \times -2 = 4x]$

$(2a-b)3a = 6a^2 - 3ab$ $\quad [2a \times 3a = 6a^2 \text{ and } -b \times 3a = -3ab]$

Example 1

Simplify:
a) $2x(x-2) - 3(x^2 - 5x - 3)$
b) $5a(a-1) - 3(a^2+1) - 4(1-a)$

Remember:
$(+) \times (+) = +$
$(-) \times (-) = +$
$(+) \times (-) = -$

Solution

a) $2x(x-2) - 3(x^2 - 5x - 3)$

$= 2x^2 - 4x - 3x^2 + 15x + 9$ $\quad \left[\begin{array}{l} \text{Multiply each term inside brackets} \\ \text{by the term in front of the brackets} \end{array}\right]$

$= 2x^2 - 3x^2 - 4x + 15x + 9$ $\quad [\text{Group like terms}]$

$= \underline{-x^2 + 11x + 9}$ $\quad [\text{Add like terms}]$

b) $5a(a-1) - 3(a^2+1) - 4(1-a)$

$= 5a^2 - 5a - 3a^2 - 3 - 4 + 4a$ $\quad \left[\begin{array}{l} \text{Multiply by using the} \\ \text{distributive law} \end{array}\right]$

$= 5a^2 - 3a^2 - 5a + 4a - 3 - 4$ $\quad [\text{Group like terms}]$

$= \underline{2a^2 - a - 7}$ $\quad [\text{Add like terms}]$

2. MULTIPLY BINOMIALS

To determine the product of two binomials, multiply every term in the first bracket by every term in the second bracket.

Example 2

Simplify: a) $(3a-2)(4a-3)$ $\quad$ b) $(2x-5y)(x+3y)$

Solution

$(3a-2)(4a-3)$ $\quad \left[\begin{array}{l} \text{Multiply every term in the first} \\ \text{bracket by every term in the} \\ \text{the second bracket} \end{array}\right]$

$= 3a(4a-3) - 2(4a-3)$ $\quad \left[\begin{array}{l} \text{Multiply each term in the second bracket} \\ \text{by the first term of the first bracket} \\ \text{then each term of the second bracket by} \\ \text{the second term of the first bracket} \end{array}\right]$

$= 12a^2 - 9a - 8a + 6$ $\quad \left[\begin{array}{l} 3a \times 4a = 12a^2 \text{ and } 3a \times -3 = -9a; \\ -2 \times 4a = -8a \text{ and } -2 \times -3 = 6 \end{array}\right]$

$\underline{= 12a^2 - 17a + 6}$ $\quad [\text{Add like terms}]$

b) $(2x-5y)(x+3y)$

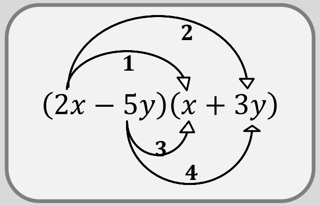
$= 2x(x+3y) - 5y(x+3y)$ $\quad \left[\begin{array}{l} \text{Each term of second bracket} \times 2x \text{ and} \\ \text{then each term of second bracket } \times -5y \end{array}\right]$

$= 2x^2 + 6xy - 5xy - 15y^2$

$= \underline{2x^2 + xy - 15y^2}$

Note the order in which the multiplication is done:

1. ***F**irst terms of each bracket*
2. ***O**uter terms of each bracket*
3. ***I**nner terms of each bracket*
4. ***L**ast terms of each bracket*



F
O
I
L

Example 3

Simplify:

a) $(3x-4)(2x+3)$
b) $-(2x-y)(5x-3y)$

Solution

$a)\quad (3x-4)(2x+3)$ $\left[\begin{matrix}FOIL:\ firsts\ (F), outers (O),\\ inners\ (I), lasts\ (L)\end{matrix}\right]$

$= 6x^2+9x-8x-12$ $\left[\begin{matrix}3x\times 2x = 6x^2;\ 3x\times 3 = 9x;\\ -4\times 2x = -8x\ ;\ -4\times 3 = 12\end{matrix}\right]$

$\underline{= 6x^2+x-12}$

$b)\quad -(2x-y)(5x-3y)$ $\left[\begin{matrix}Each\ term\ in\ 2nd\ bracket\times 2x\ ,\\ each\ term\ in\ 2nd\ bracket\times -y\end{matrix}\right]$

$= -(10x^2-6xy-5xy+3y^2)$ $\left[\begin{matrix}Keep\ the\ brackets\ around\ the\\ binomials\ while\ multiplying\end{matrix}\right]$

$= -(10x^2-11xy+3y^2)$ $[Add\ like\ terms]$

$\underline{= -10x^2+11xy-3y^2}$ $\left[\begin{matrix}Now\ multiply\ each\ term\ by -1,\\ the\ number\ in\ front\ of\ the\ brackets\end{matrix}\right]$

Example 4

Simplify: $(2x-3y)(3x-2y)-(4x+y)(x-4y)$

Solution

$(2x-3y)(3x-2y)-(4x+y)(x-4y)$ $\left[\begin{matrix}Remember: FOIL\rightarrow keep\\ brackets\ around\ the\ last\\ binomial\ while\ multiplying\end{matrix}\right]$

$= 6x^2-4xy-9xy+6y^2-(4x^2-16xy+xy-4y^2\ )$

$= 6x^2-4xy-9xy+6y^2-4x^2+16xy-xy+4y^2$ $\left[\begin{matrix}Multiply\ by\\ -1\end{matrix}\right]$

$\underline{= 2x^2+2xy+10y^2}$ $[Add\ like\ terms]$

To square a binomial means multiplying the binomial by itself. Therefore:

$(x+2)^2 = (x+2)(x+2)$ $[Write\ the\ brackets\ twice]$

$= x^2+2x+2x+4$ $\left[\begin{matrix}Multiply\ using\ FOIL: firsts,\\ outers, inners, lasts\end{matrix}\right]$

$= x^2+4x+4$

Example 5

Simplify: $(2x-y)^2$

Solution

$(2x-y)^2$
$= (2x-y)(2x-y)$ $[Write\ out\ both\ brackets]$
$= 4x^2-2xy-2xy+y^2$ $[Multiply\ brackets]$
$\underline{= 4x^2-4xy+y^2}$ $[Add\ like\ terms]$

A shorter method to square a binomial:

$(2x-y)^2$
$= 4x^2 - 4xy + y^2$
$= (2x)^2 + 2(-2x)(y) + (-y)^2$ $\left[\begin{matrix} 4x^2 = (2x)^2; \ -4xy = 2(-2x)(y) \\ and\ y^2 = (-y)^2 \end{matrix}\right]$

$= (first\ term)^2 + (2 \times\ first\ term \times second\ term) + (second\ term)^2$

You may use the method you prefer.

Example 6

Simplify: $(2x-3y)^2$

Solution

$(2x-3y)^2$
$= (2x)^2 + (2)(2x)(-3y) + (-3y)^2$ $\left[\begin{matrix} (1st\ term)^2 + (2)(1st)(2nd) \\ +(2nd\ term)^2 \end{matrix}\right]$
$\underline{= 4x^2 - 12xy + 9y^2}$

Example 7

Find the following products:

a) $(x+2)(x-2)$ b) $(2a+3)(2a-3)$

Solution

a) $(x+2)(x-2)$

$= x^2 - 2x + 2x - 4$ $\left[\begin{matrix} First\ terms, outer\ terms, \\ inner\ terms\ then\ last\ terms \end{matrix}\right]$
$\underline{= x^2 - 4}$

b) $(2a+3)(2a-3)$

$= 4a^2 - 6a + 6a - 9$ $\left[\begin{matrix} FOIL: first, outer \\ inner, last \end{matrix}\right]$
$\underline{= 4a^2 - 9}$

Note: In (a) and (b) the first terms of the two binomials are identical and the second terms are also the same but differ in sign only .

The product of two binomials of which:
the **first terms** are the same

$(a+b)(a-b) = a^2 - b^2$
$(x-y)(x+y) = x^2 - y^2$

and the **second terms** are the same, but differ in sign only,
$= (first\ term)^2 - (second\ term)^2$

Example 8

Simplify: a) $(3a+2b)(3a-2b)$ b) $(2a-\frac{1}{3})(2a+\frac{1}{3})$

Solution

a) $(3a+2b)(3a-2b)$
$= (3a)^2 - (2b)^2$ $[(first\ term)^2 - (second\ term)^2]$
$\underline{= 9a^2 - 4b^2}$

b) $(2a-\frac{1}{3})(2a+\frac{1}{3})$
$= (2a)^2 - \left(\frac{1}{3}\right)^2$ $[(first\ term)^2 - (second\ term)^2]$
$\underline{= 4a^2 - \frac{1}{9}}$

Example 9

Simplify: $2(x-2)^2 - 3(4x-3)(4x+3)$

Solution

$$2(x-2)^2 - 3(4x-3)(4x+3)$$
$$= 2(x-2)(x-2) - 3(4x-3)(4x+3) \quad [(x-2)^2 = (x-2)(x-2)]$$
$$= 2(x^2 - 2x - 2x + 4) - 3((4x)^2 - (3)^2) \; [\textit{Multiply brackets}]$$
$$= 2(x^2 - 2x - 2x + 4) - 3(16x^2 - 9)$$
$$= 2x^2 - 4x - 4x + 8 - 48x^2 + 27 \quad \begin{bmatrix} \textit{Multiply by integers} \\ \textit{in front of brackets} \end{bmatrix}$$
$$\underline{= -46x^2 - 8x + 35}$$

3. FACTORISATION

Factorisation is the reverse process of expanding brackets.
The **product** of $x(x+3) = x^2 + 3x$ while the **factors** of $x^2 + 3x = x(x+3)$.

3.1 Factorise by taking out the common factor

3.1.1 Common factor of the form $\boldsymbol{ax+bx+cx}$

The expression $\boldsymbol{ax} + b\boldsymbol{x}$ consists of two terms and $\boldsymbol{x}$ is a factor of both terms. Therefore we call $\boldsymbol{x}$ the **common** factor.

> To factorise $ax + bx$:
> Write the common factor and then a bracket: $ax + bx = x(\quad\quad)$
> Divide each term by x, the common factor: $ax + bx = x(\frac{ax}{x} + \frac{bx}{x})$
> $\underline{\therefore ax + bx = x(a+b)}$

> When you have to factorise an expression:
> ∗ First check if there is a common factor in each term.
> ∗ Write the common factor infront of a bracket.
> ∗ Divide each term of the expression by the common factor.

Example 10

Factorise the following by taking out a common factor:

a) $6x + 8y$ b) $13a - 13ab$ c) $4xy + 12yz - 8xyz$

d) $9ax^3 - 6ax^2 + 3ax$ e) $2a^2bc - 12ab^2$

Solution

a) $6x + 8y$
$\underline{= 2(3x+4y)}$ $\begin{bmatrix} 2 \textit{ is the largest common factor}: \\ 6x \div 2 = 3x \textit{ and } 8y \div 2 = 4y \end{bmatrix}$

b) $13a - 13ab$
$\underline{= 13a(1-b)}$ $\begin{bmatrix} 13a \textit{ is the common factor} \rightarrow \textit{Note}: \\ 13a \div 13 = 1 \textit{ and } -13ab \div 13a = -b \end{bmatrix}$

c) $4xy + 12yz - 8xyz$
$\underline{= 4y(x + 3z - 2xz)}$ $\begin{bmatrix} \textit{Common factor is } 4y; 4xy \div 4y = x; \\ 12yz \div 4y = 3z;\ -8xyz \div 4y = -2z \end{bmatrix}$

d) $9ax^3 - 6ax^2 + 3ax$
$\underline{= 3ax(3x^2 - 2x + 1)}$ $\begin{bmatrix} \textit{Common factor}: 3ax \rightarrow 9ax^3 \div 3ax = 3x^2; \\ -6ax^2 \div 3ax = -2x; \textit{and } 3ax \div 3ax = 1 \end{bmatrix}$

e) $2a^2bc - 12ab^2$
$\underline{= 2ab(ac - 6b)}$ $[\textit{Largest common factor} = 2ab]$
$\left[\frac{2a^2bc}{2ab} = ac \textit{ and } \frac{-12ab^2}{2ab} = -6b\right]$

Complete the first column in the table below. First try to do it on your own before you look at the answers in the second column.

1. $x^3y^2 + x^2y$ $= x^2y(\qquad\qquad)$	$= x^2y(xy + 1)$
2. $25p^2q - 20pq$ $= 5pq(\qquad\qquad)$	$= 5pq(5p - 4)$
3. $e^2fg - ef^2g - efg^2$ $= efg(\qquad\qquad)$	$= efg(e - f - g)$
4. $6x^4 - 3x^2 + 9xy$ $= \text{-----}(2x^3 - x + 3y)$	$= 3x\ (2x^3 - x + 3y)$
5. $4a^2b^2 + 2ab^2 - 6a^2b$ $= \text{-----}(2ab + b - 3a)$	$= 2ab(2ab + b - 3a)$
6. $3bx + 12by - 6bz$ $= 3b(\qquad\qquad)$	$= 3b(x + 4y - 2z)$
7. $7a^3 - 14a^2 + 28a$ $= \text{------}(a^2 - 2a + 4)$	$= 7a(a^2 - 2a + 4)$
8. $4x^4 - 2x^3y + 8x^2y^2$ $= \text{-----}(2x^2 - xy + 4y^2$	$= 2x^2(2x^2 - xy + 4y^2)$
9. $3abc^2 - 9abc - 15a^2c^2$ $= 3ac(\qquad\qquad)$	$= 3ac(bc - 3b - 5ac)$
10. $-2x^2y - 8xy^2 + 12xy$ $= -2xy(\qquad\qquad)$	$= -2xy(x + 4y - 6)$
11. $9p^2q^3z^2 - 6p^3q^2z^3$ $= \text{-----}(3q - 2pz)$	$= 3p^2q^2z^2(3q - 2pz)$
12. $-3a^2b^3c + 12ca^2b^2 - 21bca^2$ $= \text{-----}(b^2 - 4b + 7)$	$= -3a^2bc(b^2 - 4b + 7)$
13. $0{,}3x^3y^2 - 0{,}6xy + 0{,}09xy^2$ $= 0{,}3xy(\qquad\qquad)$	$= 0{,}3xy(x^2y - 2 + 0{,}3y)$

3.1.2 Common factor in the form $(\boldsymbol{a} + \boldsymbol{b})\boldsymbol{x} + (\boldsymbol{a} + \boldsymbol{b})\boldsymbol{y}$

The common factor can also be a binomial or even a trinomial. In the expression $a(\boldsymbol{x} + \boldsymbol{y}) - 3(\boldsymbol{x} + \boldsymbol{y})$ the common factor is the binomial $(\boldsymbol{x} + \boldsymbol{y})$.

Example 11

Factorise: $3x(3x - 2y) - (3x - 2y)$

Solution

$3x(3x - 2y) - (3x - 2y)$ $\qquad [Common\ factor = (3x - 2y)]$

$= (3x - 2y)(3x - 1)$ $\qquad \left[\frac{-(3x-2y)}{(3x-2y)} = -1,\ remember\ the\ -1\right]$

> $-(b - a) = -b + a = (a - b).$
> *When the sign in front of the bracket changes, the signs inside the bracket also change.*
> *In the same way* $(2y - x) = -(x - 2y)$
> *Note:* $-x$ *changes to* $+x$ *and* $+2y$ *changes to* $-2y$ *inside the bracket.*

Example 12

Factorise: $2x(3p - w) + 5y(w - 3p)$

Solution

$2x(3p - w) + 5y(w - 3p)$

$= 2x(3p - w) - 5y(-w + 3p)$ $\qquad [Change\ +5y\ to\ -5y\ and\ change\ the\ signs\ inside\ the\ bracket\ too]$

$= 2x(3p - w) - 5y(3p - w)$ $\qquad [(-w + 3p) = (3p - w)]$

$= (3p - w)(2x - 5y)$ $\qquad [Common\ factor\ is\ (3p - w)]$

Now try to complete the first column of the table on the next page.

1. $x(a+b)+2(a+b)$ $=(a+b)(\qquad)$	First try to do it on your own before you look at the answers! $=(a+b)(x+2)$
2. $x(p+q)-y(q+p)$ $=(p+q)(\qquad)$	$=(p+q)(x-y)$ Remember: $(p+q)=(q+p)$
3. $4p(2p-3q)-(2p-3q)$ $=(2p-3q)(\qquad)$	$=(2p-3q)(4p-1)$ $-(2p-3q)\div(2p-3q)=-1$
4. $3f(g-e)-g^2(g-e)$ $=(g-e)(\qquad)$	$=(g-e)(f-g^2)$
5. $9x(5a-b)+2(b-5a)$ $=9x(5a-b)-2(\qquad)$ $=(5a-b)(\qquad)$	$=9x(5a-b)-2(5a-b)$ $=(5a-b)(9x-2)$
6. $4a(b-2)+3(2-b)$ $=4a(b-2)-3(\qquad)$ $=(b-2)(\qquad)$	$=4a(b-2)-3(b-2)$ $=(b-2)(4a-3)$
7. $3b(x-y)-6c(y-x)$ $=3b(x-y)+6c(\qquad)$ $=(x-y)(\qquad)$ $=3(x-y)(\qquad)$	$=3b(x-y)+6c(x-y)$ $=(x-y)(3b+6c)$ $=3(x-y)(b+2c)$
8. $(3ab^2-9ab)-(b-3)$ $=3ab(\qquad)-(b-3)$ $=(b-3)(\qquad)$	$=3ab(b-3)-(b-3)$ $=(b-3)(3ab-1)$
9. $x(3ab-9ac)+y(3c-b)$ $=3ax(\qquad)-y(\qquad)$ $=(b-3c)(\qquad)$	$=3ax(b-3c)-y(b-3c)$ $=(b-3c)(3ax-y)$
10. $(25ab-15a)-(15b-9)$ $=5a(\qquad)-3(\qquad)$ $=(\qquad)(5a-3)$	$=5a(5b-3)-3(5b-3)$ $=(5b-3)(5a-3)$

3.2 The difference between two squares

You know that: $(x-y)(x+y)=x^2-y^2$
Reverse: $x^2-y^2=(x-y)(x+y)$

Note: x^2 is a complete square and y^2 is a complete square.
Therefore, the difference between two squares can be factorised as:

$$\left(\sqrt{\textbf{first term}}+\sqrt{\textbf{second term}}\right)\left(\sqrt{\textbf{first term}}-\sqrt{\textbf{second term}}\right)$$

Example 13

Factorise: a) $9x^2-4$ b) $-a^4b^2+9$ c) $8x-18x^3$ d) $\frac{1}{9}x^2-\frac{4}{25}$

Solution

a) $9x^2-4$ $\left[\begin{array}{l}9x^2 \textit{ and } 4 \textit{ are squares; therefore,}\\ \textit{difference between two squares}\end{array}\right]$

$\underline{=(3x-2)(3x+2)}$

b) $-a^4b^2+16$
$=(-a^4b^2+16)$
$=-(a^4b^2-16)$ [*Change the signs*]
$\underline{=-(a^2b-4)(a^2b+4)}$ $\left[\sqrt{a^4b^2}=a^2b \textit{ and } \sqrt{16}=4\right]$

c) $8x-18x^3$
$=2x(4-9x^2)$ [*Take out the common factor*]
$\underline{=2x(2-3x)(2+3x)}$ $\left[\begin{array}{l}4 \textit{ and } 9x^2 \textit{ are squares; therefore,}\\ \textit{difference between two squares}\end{array}\right]$

d) $\frac{1}{9}x^2-\frac{4}{25}$
$\underline{=(\frac{1}{3}x-\frac{2}{5})(\frac{1}{3}x+\frac{2}{5})}$ $\left[(\sqrt{\frac{1}{9}x^2}-\sqrt{\frac{4}{25}})(\sqrt{\frac{1}{9}x^2}+\sqrt{\frac{4}{25}}\right]$

Now complete the first column of the table below. Remember, try to do it on your own before you look at the answers in the second column!

1. $x^4 - 25y^2$ $= (\qquad)(\qquad)$	$= (x^2 - 5y)(x^2 + 5y)$
2. $25p^4 - q^8r^6$ $= (\qquad)(\qquad)$	$= (5p^2 - q^4r^3)(5p^2 + q^4r^3)$
3. $e^5 - ef^2$ $= e(\qquad)$ $= e(\qquad)(\qquad)$	$= e(e^4 - f^2)$ $= e(e^2 - f)(e^2 + f)$
4. $8x^4 - 18y^2$ $= 2(\qquad)$ $= 2(\qquad)(\qquad)$	$= 2(4x^4 - 9y^2)$ $= 2(2x^2 - 3y)(2x^2 + 3y)$
5. $36a^2 - 36b^2$ $= 36(\qquad)$ $= 36(\qquad)(\qquad)$	$= 36(a^2 - b^2)$ $= 36(a - b)(a + b)$
6. $3bx^8 - 12by^6$ $= 3b(\qquad)$ $= 3b(\qquad)(\qquad)$	$= 3b(x^8 - 4y^6)$ $= 3b(x^4 - 2y^3)$
7. $a^2 - \frac{1}{9}$ $= (\qquad)(\qquad)$	$= (a - \frac{1}{3})(a + \frac{1}{3})$
8. $-\frac{16}{a^2} + 1$ $= -(\qquad)$ $= -(\qquad)(\qquad)$	$= -(\frac{16}{a^2} - 1)$ $= -(\frac{4}{a} - 1)(\frac{4}{a} + 1)$
9. $5 - 5a^4$ $= 5(\qquad)$ $= 5(\qquad)(\qquad)$ $= 5(\qquad)(\qquad)(\qquad)$	$= 5(1 - a^4)$ $= 5(1 - a^2)(1 + a^2)$ $= 5(1 - a)(1 + a)(1 + a^2)$

3.4 Factors of the trinomial in the form $x^2 + bx + c$

You know that $(x + 4)(x + 3) = x^2 + 3x + 4x + 12 = x^2 + 7x + 12$. The product of two binomials is usually a trinomial and therefore, the factors of a trinomial will be two binomials.

$$(x + a)(x + b) = x^2 + ax + bx + ab$$
$$= x^2 + x(a + b) + ab$$
$$= x^2 + x(\text{sum of } a \text{ and } b) + (\text{product of } a \text{ and } b)$$

$$(x + 3)(x + 5) = x^2 + 8x + 15$$
$$= x^2 + x(3 + 5) + (3 \times 5)$$

To determine the factors of $x^2 + \mathbf{b}x + \mathbf{c}$, *find 2 factors of* $\mathbf{c}$ *which add up to* $\mathbf{b}$

To determine the factors of $x^2 - \mathbf{5}x + \mathbf{6}$, *find two factors of* $\mathbf{6}$ *which add up to* $-\mathbf{5}$

If the last term is positive, the signs inside the brackets will be the same .

$x^2 + 5x + 6 = (x + 2)(x + 3)$ $\qquad$ $x^2 - 5x + 6 = (x - 2)(x - 3)$

The signs inside the brackets will be the same as the sign of the middle term.

If the last term is negative, the signs inside the brackets will differ.

$x^2 + 5x - 6 = (x + 5)(x - 1)$ $\qquad$ $x^2 - 4x - 8 = (x - 4)(x + 2)$

Example 14

Factorise fully:

a) $x^2 + 9x + 18$ $\qquad$ b) $x^2 - 8x + 15$
c) $p^2 + p - 30$ $\qquad$ d) $y^2 - 5y - 24$

Solution

a) $x^2 + 9x + 18$

Sum of factors: +9

1×18
2×9
3×6

$\left[\begin{array}{l}\textit{The last term is } +\textit{, signs inside brackets}\\ \textit{will be the same, middleterm is } +,\\ \textit{the signs inside the brackets both } +\end{array}\right]$

$= (x + 6)(x + 3)$

$\left[\begin{array}{l}3 \times 6 = 18 \textit{ and } 3 + 5 = 18; \textit{therefore,}\\ 3 \textit{ and } 6 \textit{ are the correct combination}\end{array}\right]$

b) $x^2 - 8x + 15$

Sum of factors: -8

1×15
3×5

$\left[\begin{array}{l}\textit{The last term is } +\textit{, signs inside brackets}\\ \textit{will be the same, middleterm is } -,\\ \textit{the signs inside the brackets both } -\end{array}\right]$

$= (x - 3)(x - 5)$

$\left[\begin{array}{l}3 \times 5 = 15 \textit{ and } 3 + 5 = 8 \textit{ but the}\\ \textit{signs must be negative because the}\\ \textit{sum of the factors must be } -8\end{array}\right]$

c) $p^2 + p - 30$

$\left[\begin{array}{l}\textit{Last term negative, signs}\\ \textit{inside brackets differ}\end{array}\right]$

Sum of factors: +1

1×30
2×15
3×10
5×6

$= (x + 6)(x - 5)$

$\left[\begin{array}{l}5 \times -6 \textit{ are the correct combination;}\\ +6 - 5 = 1 \textit{ and } 5 \times -6 = -30\end{array}\right]$

d) $y^2 - 5y - 24$

$[\textit{Last term } -, \textit{ signs inside brackets differ}]$

$1 \times 24;\ 2 \times 12;\ (3 \times 8);\ 4 \times 6$

$= (y + 3)(y - 8)$

$\left[\begin{array}{l}3 \times -8 \textit{ are the correct combination;}\\ +3 - 8 = -5 \textit{ and } 3 \times -8 = -24\end{array}\right]$

Once again, complete the first column. Write the factors of the last term in the second column and then find the correct combination.

1. $x^2 - 11x + 24$ $= (\ x - 8\ )(\ x - 3)$	1×24 ; 2×12 3×8; 4×6	$3 + 8 = 11$ The signs are both negative $-$
2. $a^2 + 8a + 12$ $= (\qquad)(\qquad)$		$= (a + 6)(a + 2)$
3. $p^2 - 5p - 14$ $= (\qquad)(\qquad)$		$= (p - 7)(p + 2)$
4. $m^2 + 4m - 12$ $= (\qquad)(\qquad)$		$= (m + 6)(m - 2)$
5. $x^2 + 11x + 28$ $= (\qquad)(\qquad)$		$= (x + 7)(x + 4)$
6. $3x^2 - 15x - 18$ $= 3(\qquad\qquad)$ $= 3(\qquad)(\qquad)$		$= 3(x^2 - 5x - 6)$ $= 3(x - 6)(x + 1)$
7. $x^2 - 3x - 28$ $= (\qquad)(\qquad)$		$= (x - 7)(x + 4)$
8. $6a^2 - 48a - 120$ $= 6(\qquad\qquad)$ $= (\qquad)(\qquad)$		$= 6(a^2 - 8a - 20)$ $= 6(a - 10)(a + 2)$
9. $y^2 - 8y + 7$ $= (\qquad)(\qquad)$		$= (y - 7)(y - 1)$
10. $x^2 - 8x + 16$ $= (\qquad)(\qquad)$		$= x - 4)(x - 4)$
11. $x^2 + 4x - 21$ $= (\qquad)(\qquad)$		$= (x + 7)(x - 3)$
12. $3m^2 - 27m + 24$ $= 3(\qquad\qquad)$ $= 3(\qquad)(\qquad)$		$= 3(m^2 - 9m + 8$ $= 3(m - 8)(m - 1)$

4. USE FACTORISATION TO SIMPLIFY FRACTIONS

Before simplifying algebraic fractions, first factorise numerators and denominators **fully.**

For example: $\frac{3x-6}{3x-9} = \frac{3(x-2)}{3(x-3)} = \frac{(x-2)}{(x-3)}$ **but:** $\frac{3x-6}{3x-9} \neq \frac{-6}{-9} \neq \frac{2}{3}$

Like terms in numerator and denominator may not be cancelled, only like factors in numerator and denominator.

Example 15

Simplify the following:

a) $\frac{x^2-2x-15}{2x^2-50}$ b) $\frac{2a-2ab}{ab-a}$

c) $\frac{5a+5b+5c}{5a-10} \times \frac{a^2-3a-10}{pa+pb+pc}$ d) $\frac{x^2+2x}{2x^2-6x} \div \frac{x^2-4}{2x-4}$

Solution

a) $\frac{x^2-2x-15}{2x^2-50}$

$= \frac{(x-5)(x+3)}{2(x^2-25)}$ $\left[\begin{array}{l} \text{Factorise numerator} \rightarrow -5 \times 3 = 15 \\ \text{and } -5+3=-2 \text{ thus } (x-5)(x+3) \\ 2 \text{ is a common factor in denominator} \end{array}\right]$

$= \frac{(x-5)(x+3)}{2(x-5)(x+5)}$ $\left[\begin{array}{l} \text{Factorise denominator fully} \rightarrow \text{difference} \\ \text{between 2 squares, cancel common factors} \end{array}\right]$

$= \underline{\frac{(x+3)}{2(x+5)}}$

b) $\frac{2a-2ab}{ab-a}$

$= \frac{2a(1-b)}{a(b-1)}$ [Factorise numerator and denominator]

$= \frac{-2a(b-1)}{a(b-1)}$ $\left[\begin{array}{l} \text{Change } (1-b) \text{ to } (b-1) \text{ and} \\ \text{cancel common factors} \end{array}\right]$

$\underline{= -2}$

c) $\frac{5a+5b+5c}{5a-10} \times \frac{a^2+3a-10}{pa+pb+pc}$

$= \frac{5(a+b+c)}{5(a-2)} \times \frac{(a-2)(a+5)}{p(a+b+c)}$ $\left[\begin{array}{l} \text{Factorise all numerators and denominators} \\ \text{and cancel common factors} \end{array}\right]$

$= \underline{\frac{(a+5)}{p}}$

d) $\frac{x^2+2x}{2x^2-6x} \div \frac{x^2-4}{2x-4}$

$= \frac{x^2+2x}{2x^2-6x} \times \frac{2x-4}{x^2-4}$ [Change $\div$ to $\times$ and reverse fraction]

$= \frac{x(x+2)}{2x(x-3)} \times \frac{2(x-2)}{(x-2)(x+2)}$ $\left[\begin{array}{l} \text{Factorise all numerators and denominators} \\ \text{and cancel common factors} \end{array}\right]$

$= \underline{\frac{1}{x-3}}$

Example 16

Simplify:

a) $\frac{x^2-3x-4}{3x^2-12x}$ b) $\frac{x(3a-b)+y(3a-b)}{9a^2(x+y)-b^2(x+y)}$

Solution

a) $\frac{x^2-3x-4}{3x^2-12x}$

$= \frac{(x-4)(x+1)}{3x(x-4)}$ $\left[\begin{array}{l} Factorise\ numerator\ and\ denomonator; \\ cancel\ the\ common\ factor\ (x-4) \end{array}\right]$

$= \frac{(x+1)}{3x}$

b) $\frac{x(3a-b)+y(3a-b)}{9a^2(x+y)-b^2(x+y)}$

$= \frac{(3a-b)(x+y)}{(x+y)(9a^2-b^2)}$ $\left[\begin{array}{l} Factorise\ numerator\ and\ denominator; \\ Note: 9a^2-b^2\ is\ not\ factorised\ fully \end{array}\right]$

$= \frac{(3a-b)(x+y)}{(x+y)(3a-b)(3a+b)}$ $\left[\begin{array}{l} Factorise\ denominator\ fully\ and \\ cancel\ common\ factors \end{array}\right]$

$= \frac{1}{(3a+b)}$

Multiply a binomial by a binomial:

$(x-2y)(3x-y) = 3x^2 - xy - 6xy + 2y^2$

$\underline{= 3x^2 - 7xy + 2y^2}$

Square a binomial:

Remember!

$(2x-y)^2 = (2x-y)(2x-y)$ $\left[\begin{array}{l} (2x-y)^2 \neq 4x^2 - y^2\ maar \\ (2x-y)^2 = (2x-y)(2x-y) \end{array}\right]$

$= 4x^2 - 2xy - 2xy + y^2$

$\underline{= 4x^2 - 4xy + y^2}$

FACTORISATION

Common factor: $2x^2 - x$

$= x(2x-1)$ $[Common\ factor \rightarrow x]$

Common bracket: $2x(x-y) - y(x-y)$

$= (x-y)(2x-y)$ $[Common\ factor \rightarrow (x-y)]$

Switch around: $x(x-y) + 3y(y-x)$

$= x(x-y) - 3y(x-y)$ $\left[\begin{array}{l} Change\ +\ between\ brackets \\ to\ -; (y-x) becomes\ (x-y) \end{array}\right]$

$= (x-y)(x-3y)$ $[Common\ factor \rightarrow (x-y)\ ]$

Difference between two squares

∗ The expression consists of two terms only.
∗ The terms are separated by a minus sign.
∗ Each term is a complete square.

$x^2 - 9y^2 \underline{= (x-3y)(x+3y)}$ $[(\sqrt{x^2} - \sqrt{9y^2})(\sqrt{x^2} - \sqrt{9y^2})]$

$(x-y)^2 - 25 = [(x-y)-5][(x-y)+25]$

$\underline{= (x-y-5)(x-y+5)}$

We cannot factorise the sum of two squares, e.g., $x^2 + y^2$.

Trinomial

$x^2 + 8x + 15$ $\left[\begin{array}{l} Last\ term\ +,\ signs\ the\ same; \\ middle\ term\ +,\ both\ signs\ + \end{array}\right]$

$\underline{= (x+3)(x+5)}$ $[3 \times 5 = 15\ and\ 3 + 5 = 8]$

$x^2 - 9x + 18$ $\left[\begin{array}{l} Last\ term\ +,\ signs\ the\ same; \\ middle\ term\ -,\ both\ signs\ - \end{array}\right]$

$\underline{= (x-3)(x-6)}$ $[-3 \times -6 = -18\ and\ -3 - 6 = -9]$

$x^2 - 5x - 14$ $[Last\ term\ +,\ signs\ differ]$

$\underline{= (x+2)(x-7)}$ $[2 \times -7 = -14\ and\ 2 - 7 = -5]$